# Spreadsheet Math

## A Powerful Tool for the Practice of Mathematics

NCTM Boston 2015

April 17, 2015

**Art Bardige** **Peter Mili** **Ryan McQuade**

# Agenda

- Introduction
- What if…
- Labs
- Questions and Comments

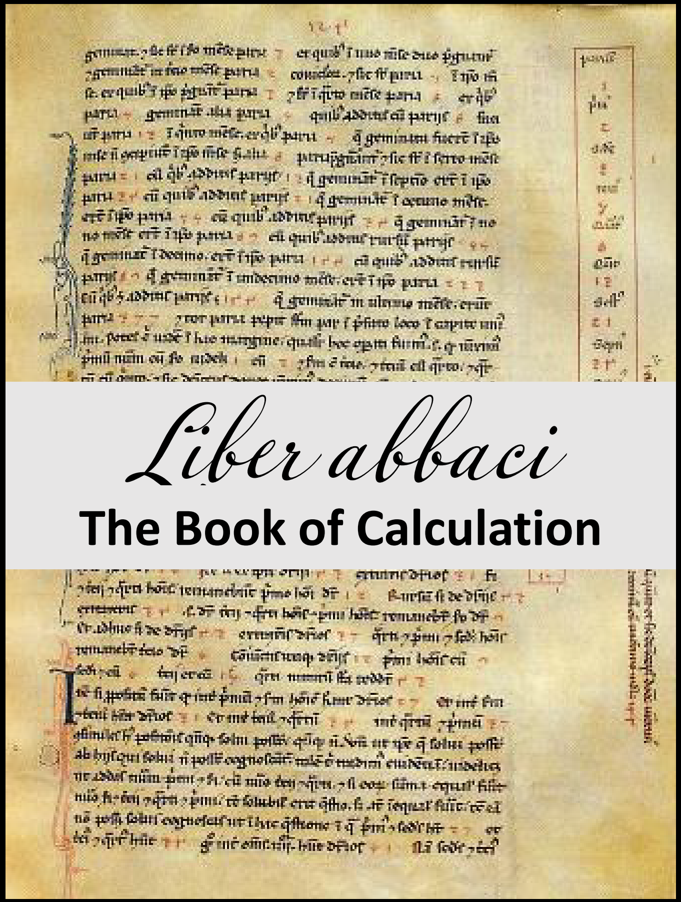
What if… we started from scratch to design a math curriculum for 21st century students using 21st century tools?

Our math curriculum today was designed…

*Leonardo of Pisa (c.1170-1250)*

In the year 1202 when Leonardo of Pisa…

Reinvented
the mathematics used
by merchants
with this book

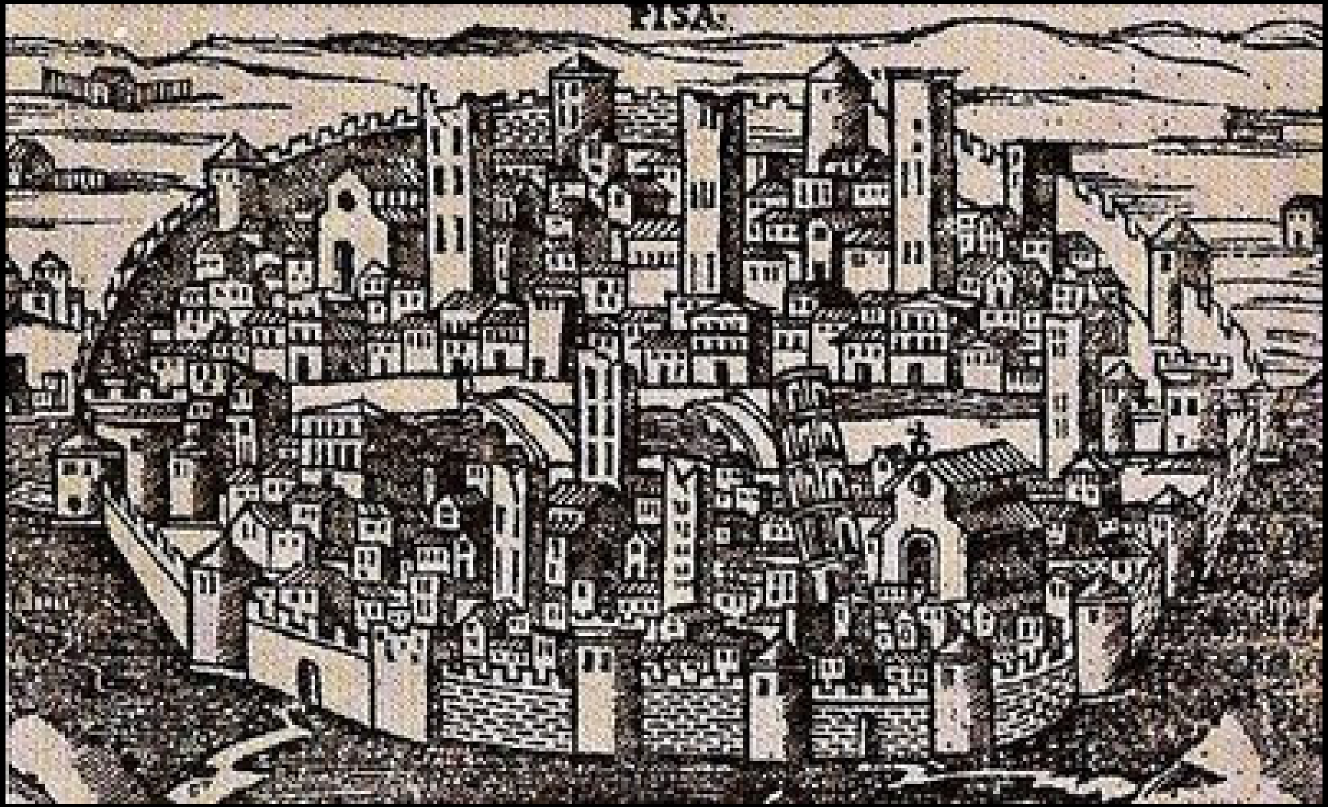



*Liber abbaci (1202)*

He was born
in Pisa

At the same time as the Leaning Tower

When Pisa was a great trading city



As a boy Leonardo followed his father, a “public official” and trader to Algeria

Where he was tutored in
Arabic arithmetic and algebra

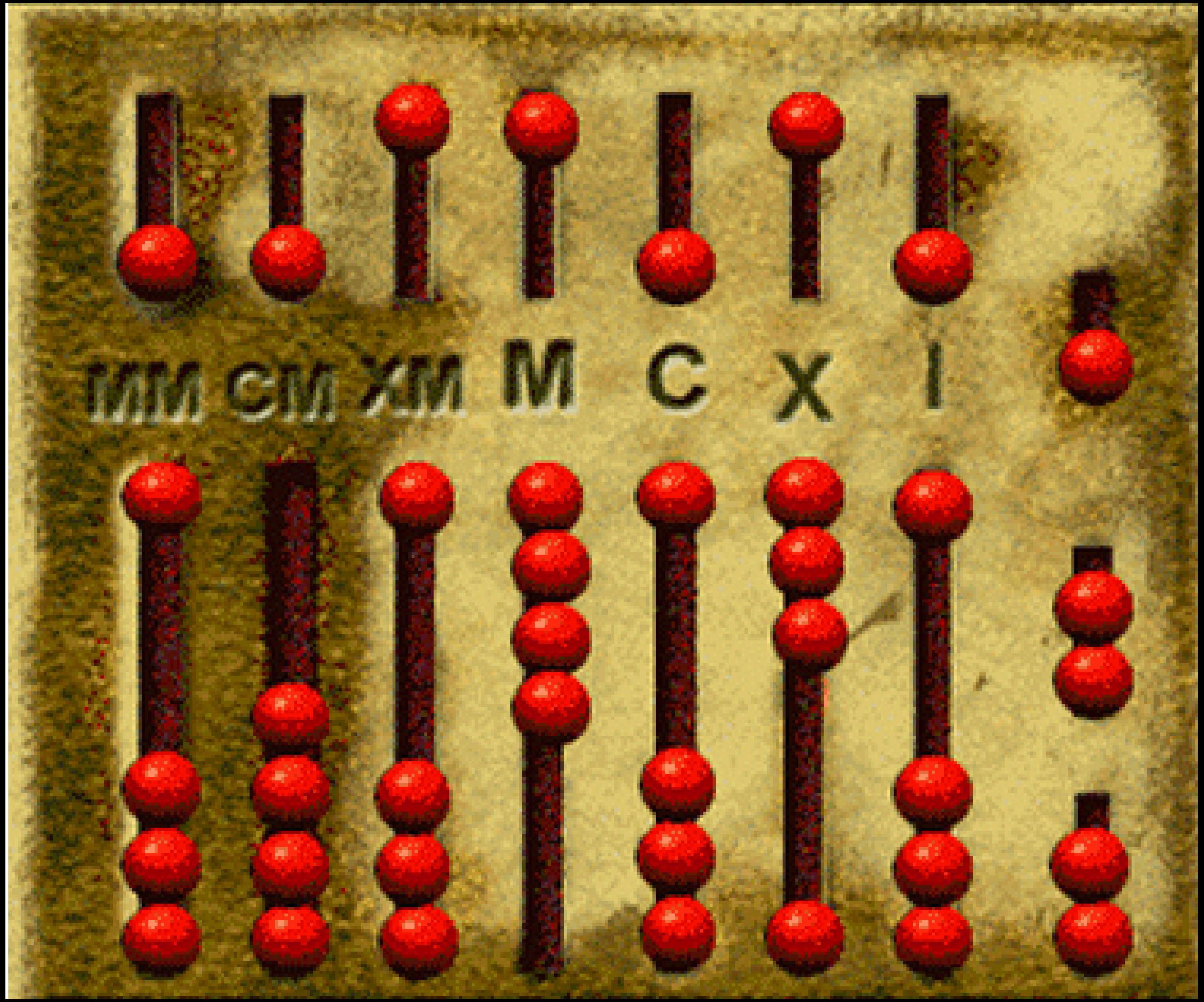

*The Compendious Book on Calculation by Completion and Balancing*
*al Khwarizmi*

Both academic subjects…

*Scholars at an Abbasid library, Baghdad (1237)*

…not used by
medieval merchants

Who computed in Roman math on an abacus

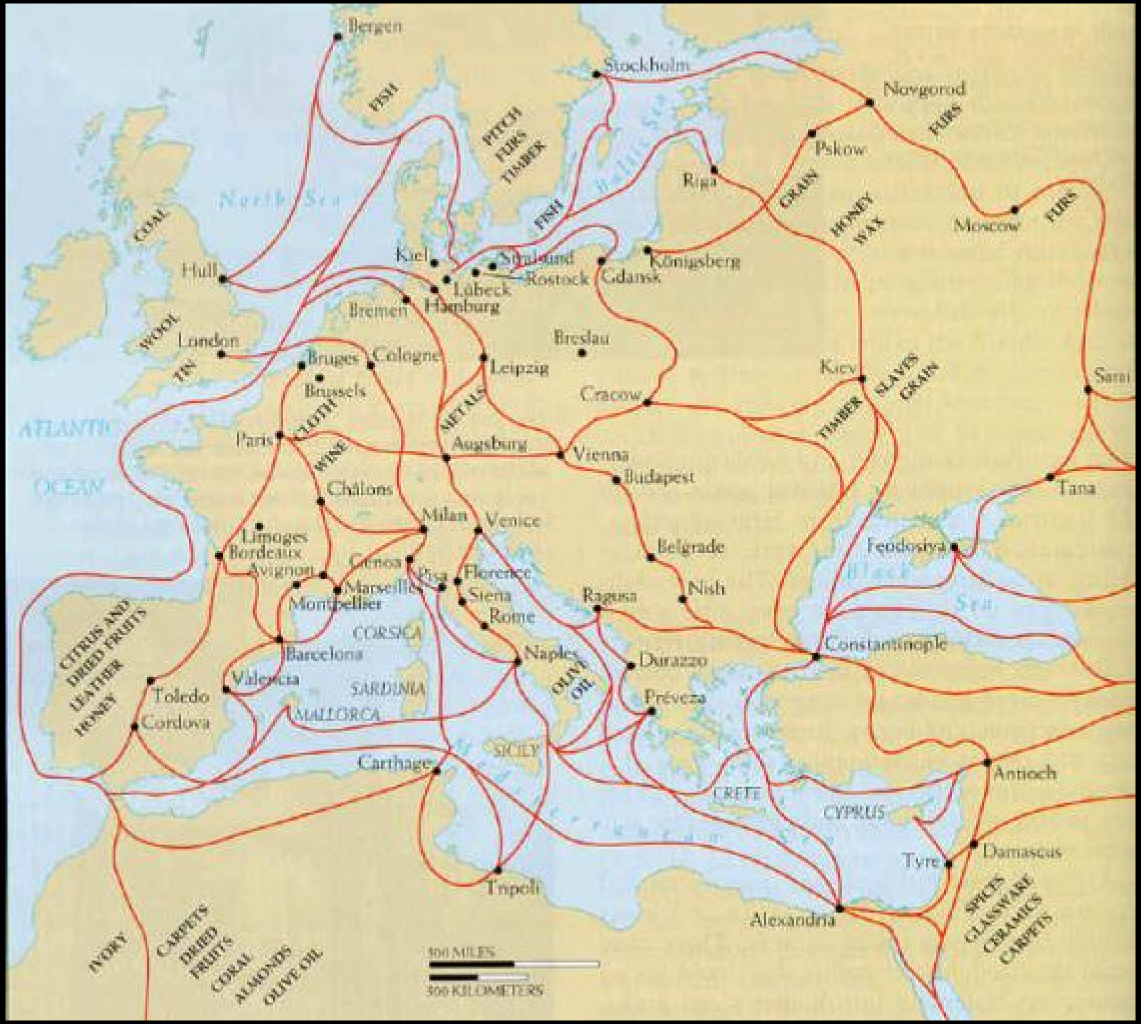


# Good enough for the Roman Empire

*"None of the cities should be allowed to have its own separate coinage or a system of weights and measures; they should all be required to use ours."*

Dio Cassius 235AD

But not for trade between Medieval city-states, each with its own…



…weights, measures, and money

Requiring multiplication, division and the solving of complex ratio and proportion problems

| | |
|---|---|
| **CLXVII** | I |
| CLXVIICLXVII | II |
| CLXVIICLXVIICLXVIICLXVII | IV |
| CLXVIICLXVIICLXVIICLXVII<br>CLXVIICLXVIICLXVIICLXVII | VIII |
| CLXVII<br>CLXVIICLXVIICLXVIICLXVII<br>CLXVIICLXVIICLXVIICLXVII | I+VIII=IX |

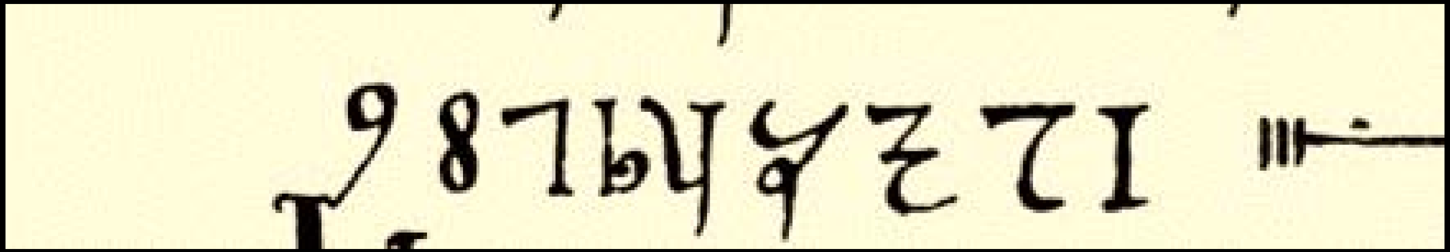
*Roman multiplication by Doubling (167 x 9 = )*

Leonardo returned to Pisa to write "arithmetic necessary to merchants"

The Liber Abaci or Book of Calculation is by Leonardo Pisano Bigollo or Fibonacci, considered by many as one of the most talented western mathematicians of the Middle Ages. Featured in a rare 15th century manuscript estimated at between $120,000-180,000 (£75,000 – 110,000). Photo: by Bonhams.

*Liber abbaci*

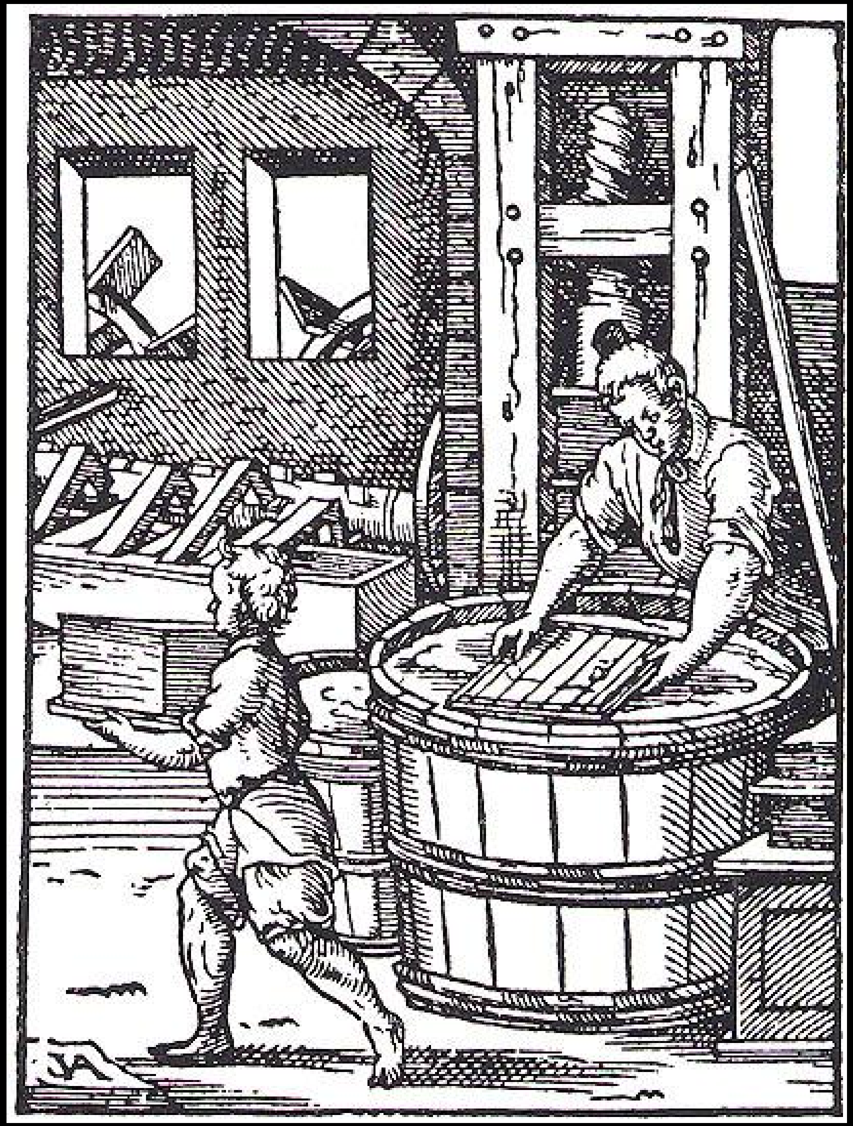
Based on Indian numerals, place value and…



al Khwarizmi's “algorithmic” procedures in arithmetic and algebra

*al Khwarizmi (c780-c850)*

Using the new technology…
paper



*Paper introduced to Europe c. 1100*

Leonardo’s (algorist) math gradually become symbolic and made

*Algorist vs. Abacist (woodcut 1504)*

Leonardo’s (algorist) math gradually become symbolic and made Roman (abacist) math...

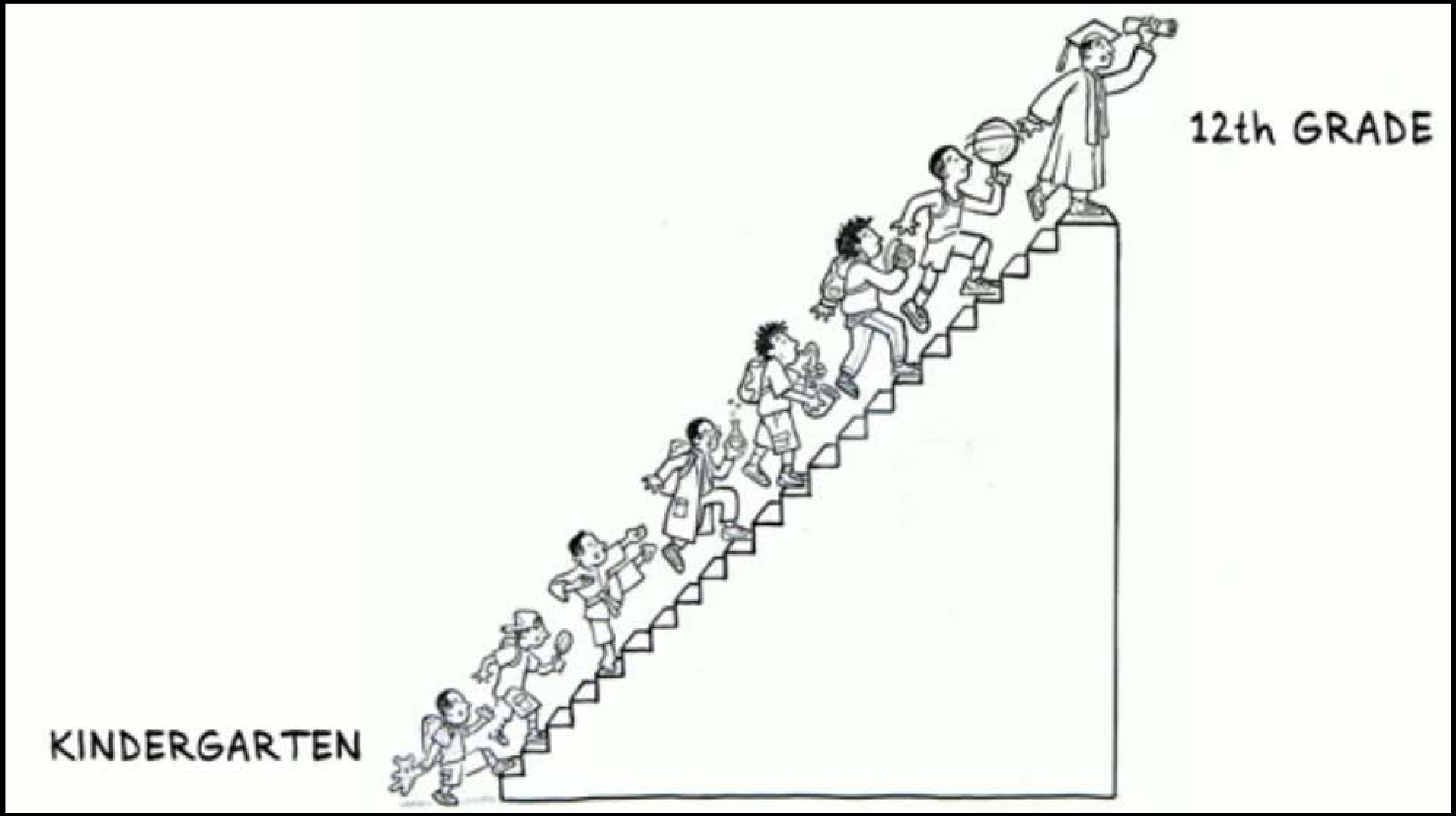

*Algorist vs. Abacist (woodcut 1504)*

# By the 17th century Leonardo's table of contents…

1. *On the recognition of the nine Indian figures and how all numbers are written with them. (place value)*
2. *On the multiplication of whole numbers*
3. *On the addition of them, one to another*
4. *On the subtraction of lesser numbers from greater numbers*
5. *On the division of integral numbers*
6. *On the multiplication of integral numbers with fractions*
7. *On the addition and subtraction and division of numbers and fractions and the reduction of parts to a single part*
8. *On the buying and selling of commercial things (ratio & proportion)*
9. *On the barter of commercial things (rate)*
10. *On companies made among parties (percents)*
11. *On the alloying of money (mixture problems)*
12. *On the solutions of many problems (Fibonacci sequence)*
13. *On the rule of elchataym by which problems of false position are solved. (solving linear equations)*
14. *On the finding of square and cube roots, on binomials and their roots.*
15. *On the pertinent rules of geometric proportions*

2013

This became the curriculum staircase we all know so well




*Video on Common Core Math Standards*

Defined by the difficulty level of the algorithms

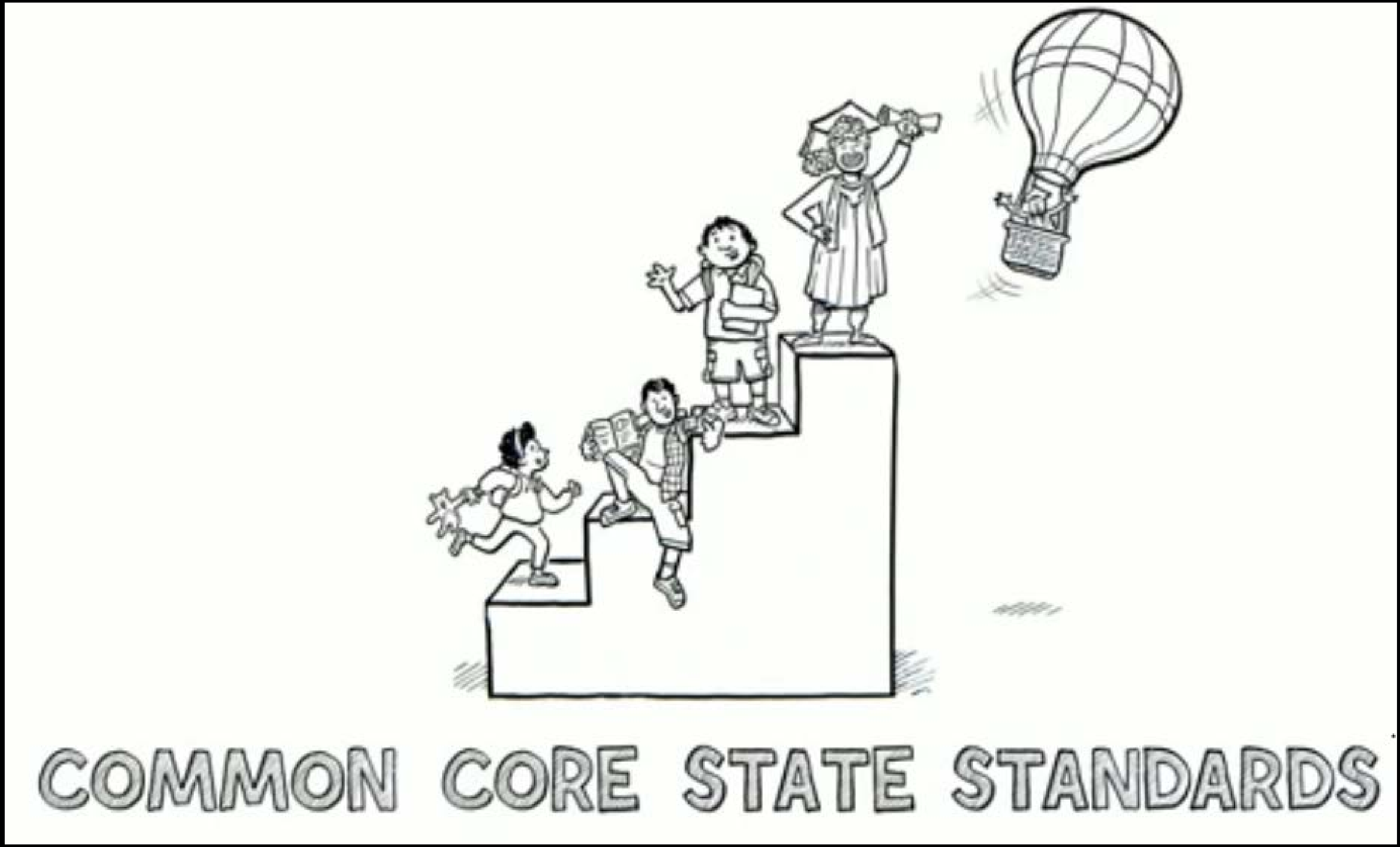
$$23\overline{)4381}$$

$$\begin{array}{r} 847 \\ \underline{\times\ 74} \end{array}$$

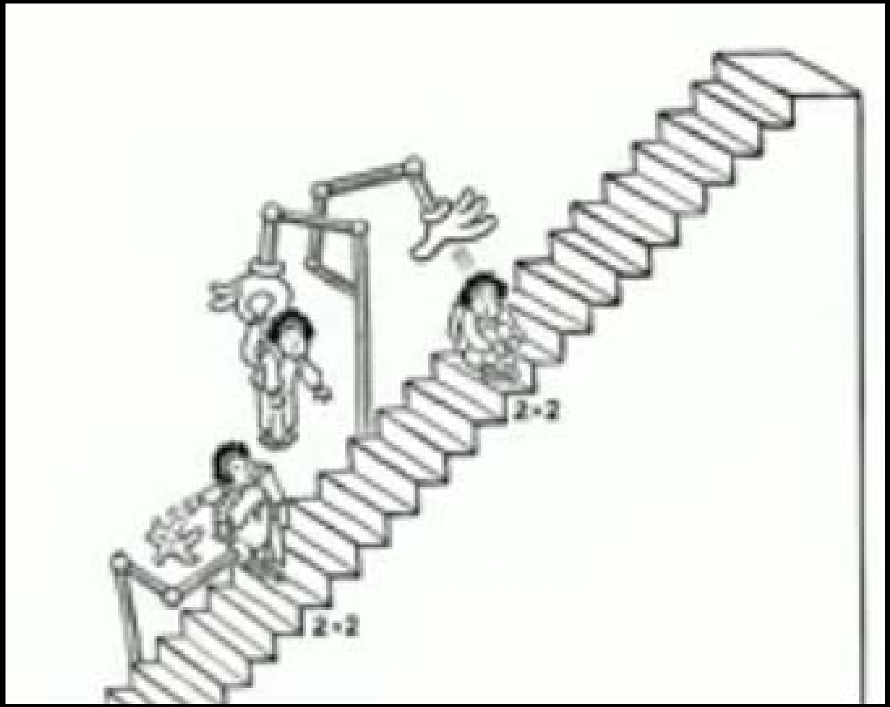
$$\begin{array}{r} 647 \\ \underline{-\ 49} \end{array}$$

$$\begin{array}{r} 5280 \\ \underline{+173} \end{array}$$

Every student must climb today!



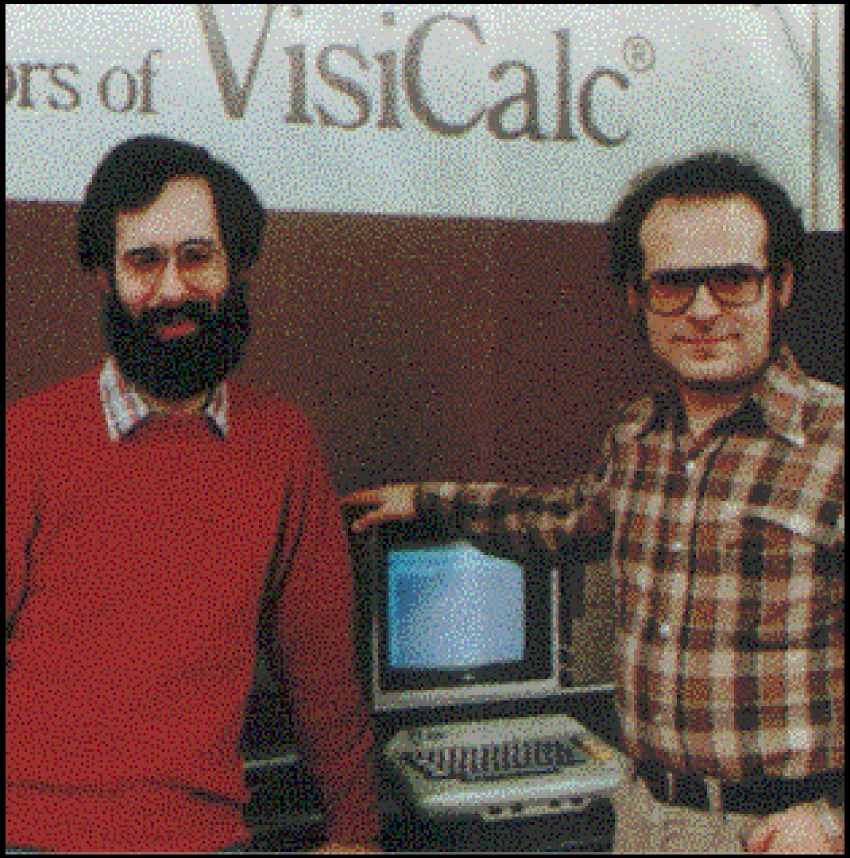
Yet, so many
“fall behind”
and
fail



To understand what we should do about this great problem we must first know what 21st century students need

For in 1979 a new technology reinvented the mathematics of business

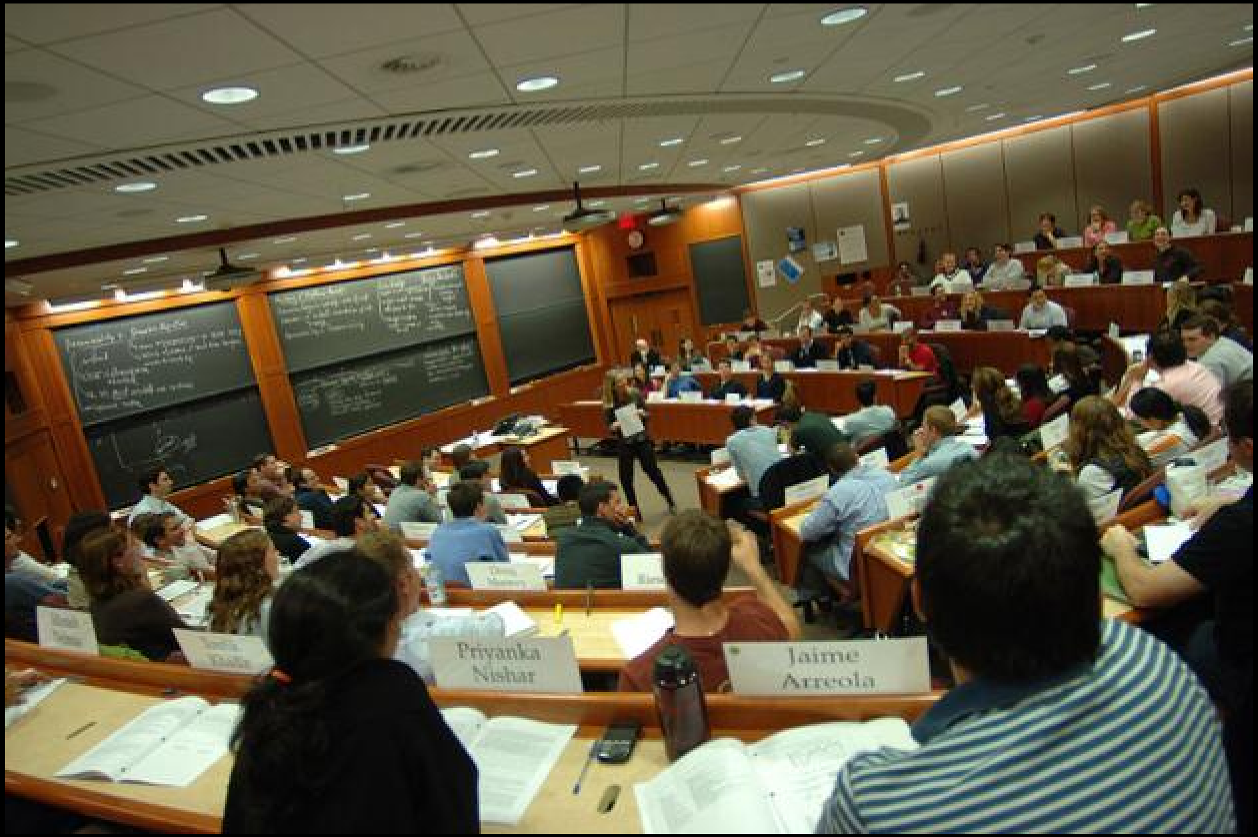
Dan Bricklin



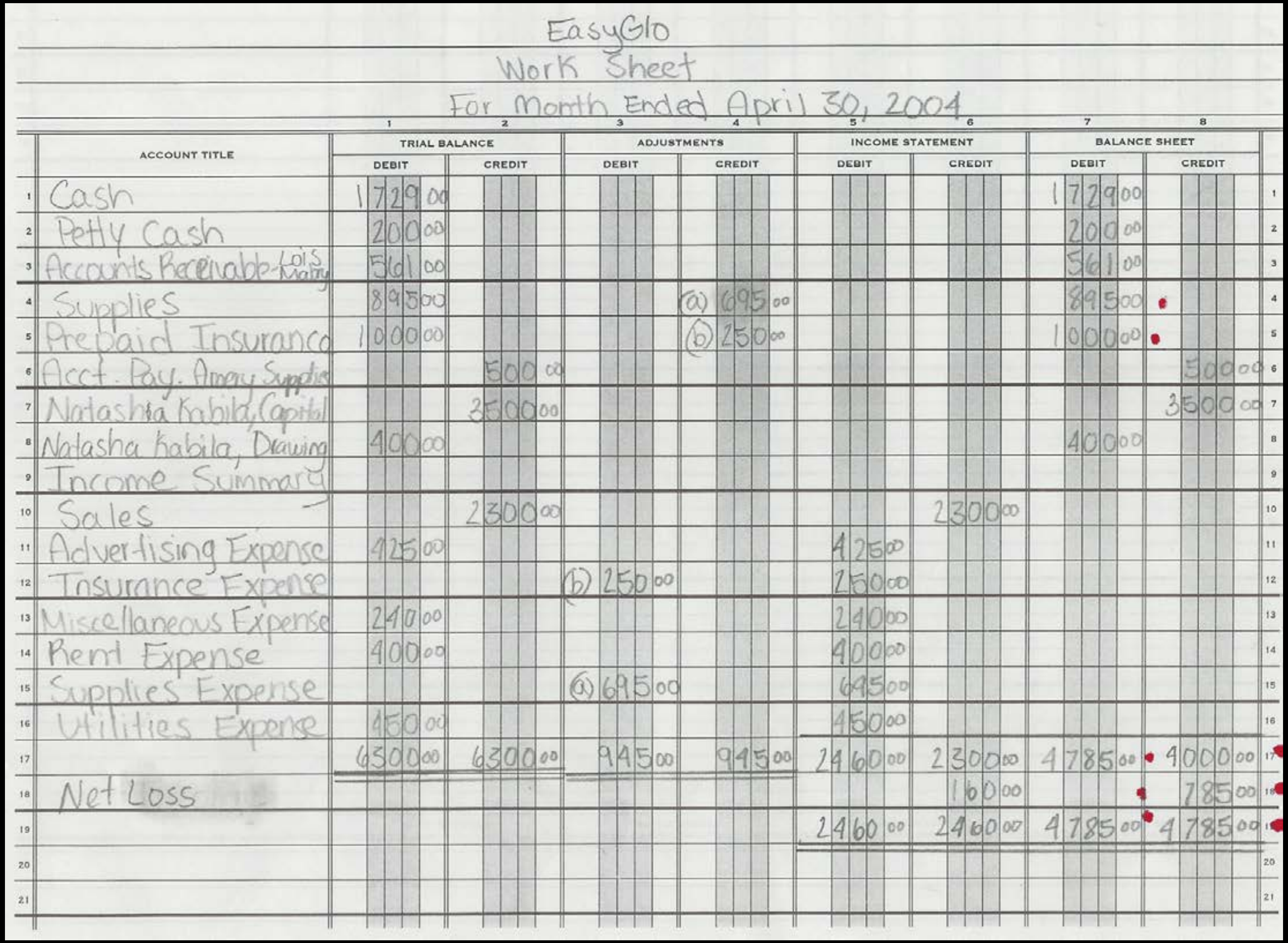

*Dan Bricklin & Bob Frankston*

A Harvard Business School student

# Working on case studies



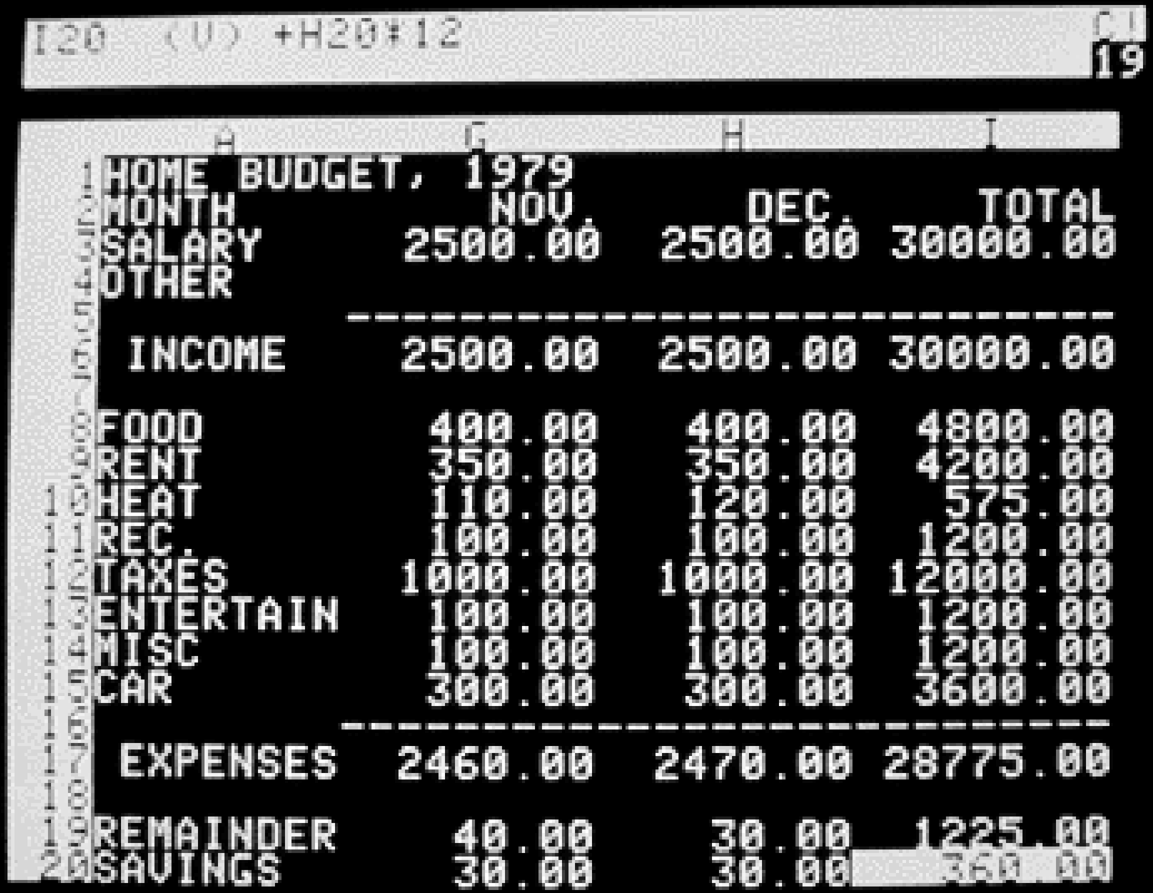
Wanted technology to enable him to ask "What if…"



EasyGlo
Work Sheet
For Month Ended April 30, 2004

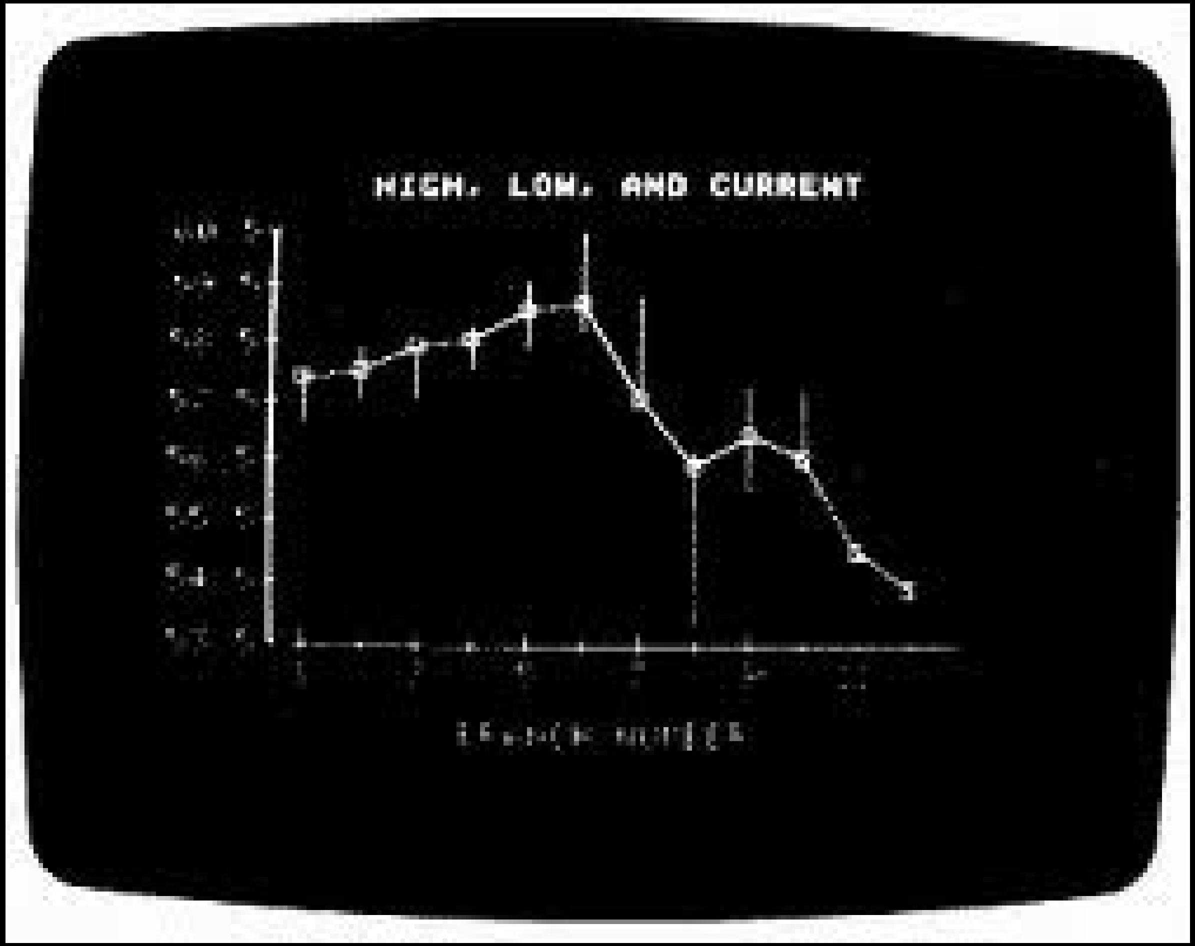
| | ACCOUNT TITLE | TRIAL BALANCE DEBIT (1) | TRIAL BALANCE CREDIT (2) | ADJUSTMENTS DEBIT (3) | ADJUSTMENTS CREDIT (4) | INCOME STATEMENT DEBIT (5) | INCOME STATEMENT CREDIT (6) | BALANCE SHEET DEBIT (7) | BALANCE SHEET CREDIT (8) | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Cash | 1729 00 | | | | | | 1729 00 | | 1 |
| 2 | Petty Cash | 200 00 | | | | | | 200 00 | | 2 |
| 3 | Accounts Receivable-Lois Malry | 561 00 | | | | | | 561 00 | | 3 |
| 4 | Supplies | 895 00 | | | (a) 695 00 | | | 895 00 | | 4 |
| 5 | Prepaid Insurance | 1000 00 | | | (b) 250 00 | | | 1000 00 | | 5 |
| 6 | Acct. Pay. Amery Supplies | | 500 00 | | | | | | 500 00 | 6 |
| 7 | Natashia Kabila, Capital | | 3500 00 | | | | | | 3500 00 | 7 |
| 8 | Natasha Kabila, Drawing | 400 00 | | | | | | 400 00 | | 8 |
| 9 | Income Summary | | | | | | | | | 9 |
| 10 | Sales | | 2300 00 | | | | 2300 00 | | | 10 |
| 11 | Advertising Expense | 425 00 | | | | 425 00 | | | | 11 |
| 12 | Insurance Expense | | | (b) 250 00 | | 250 00 | | | | 12 |
| 13 | Miscellaneous Expense | 240 00 | | | | 240 00 | | | | 13 |
| 14 | Rent Expense | 400 00 | | | | 400 00 | | | | 14 |
| 15 | Supplies Expense | | | (a) 695 00 | | 695 00 | | | | 15 |
| 16 | Utilities Expense | 450 00 | | | | 450 00 | | | | 16 |
| 17 | | 6300 00 | 6300 00 | 945 00 | 945 00 | 2460 00 | 2300 00 | 4785 00 | 4000 00 | 17 |
| 18 | Net Loss | | | | | | 160 00 | | 785 00 | 18 |
| 19 | | | | | | 2460 00 | 2460 00 | 4785 00 | 4785 00 | 19 |
| 20 | | | | | | | | | | 20 |
| 21 | | | | | | | | | | 21 |

So he and Bob Frankston invented the spreadsheet



*VisiCalc the Visible Calculator*

Mitch Kapor added graphs



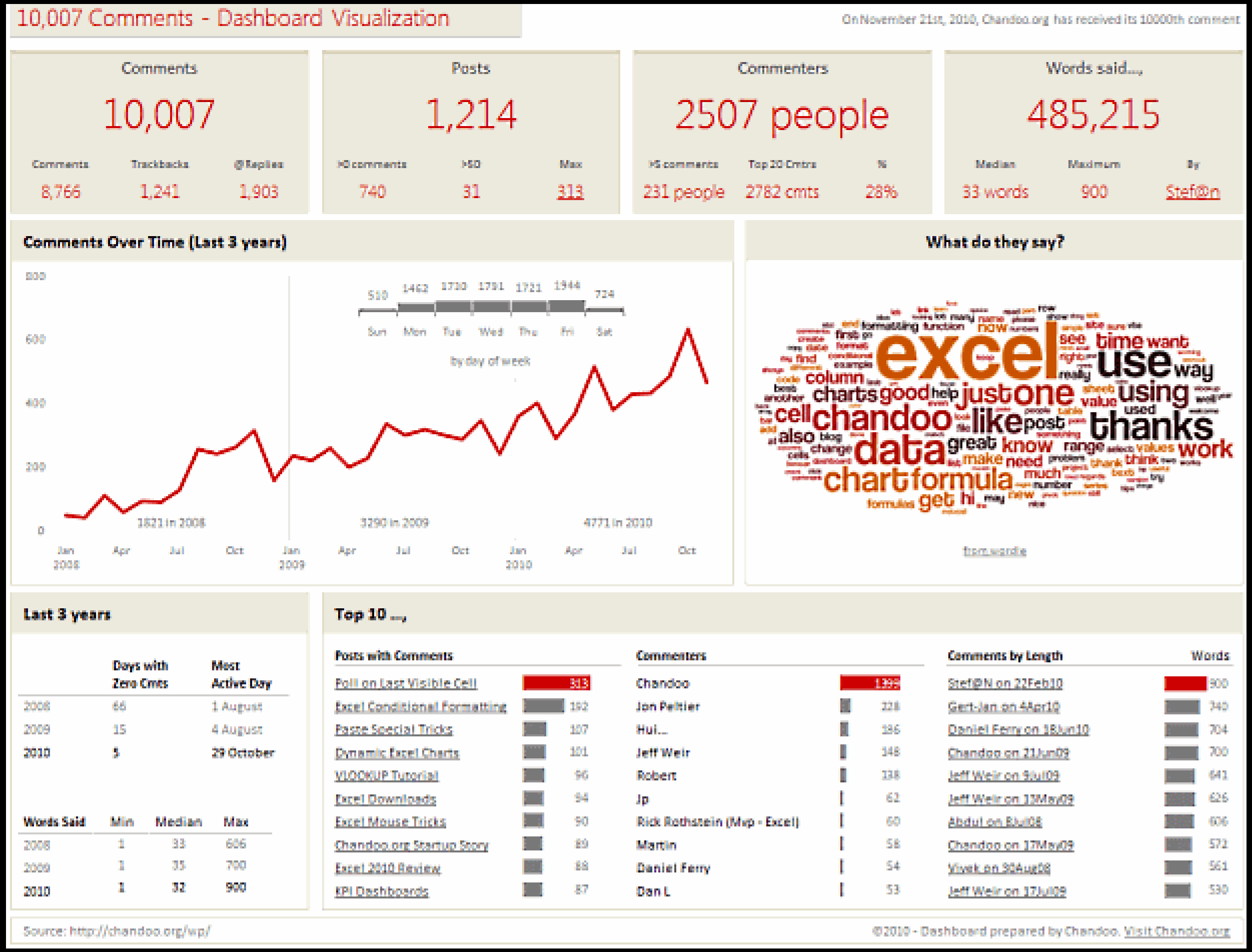

*VisiPlot*

And then a
database


*Mitch and Lotus 123*

Putting a PC on every business desk with…

# …a spreadsheet inside



To enable business people to now ask…

To enable
business people
to now ask…

*What if…*

Not only what is

*What if…*

Making Leonardo's math…

$\underline{5280 + 1732 =}$

$$\begin{array}{r} 647 \\ \underline{*44} \end{array}$$

$$6\frac{2}{3} - \frac{1}{8} =$$

$$\frac{5}{6} / \frac{-7}{12} =$$

$$\frac{438}{25} = 17\,r14$$

$$\sqrt[3]{64 + \sqrt{81}}$$

$$a^2 + b^2 = c^2$$

$$A = \pi r^2$$

$$3x - 7 = 11$$

$$\frac{16}{9} = \frac{6}{x}$$

$$\frac{10}{7}x + 1 = \frac{3}{2}x - 8$$

$$2x^2 - 8x + 14$$

$$(15x^2 + 8x - 4)/(3x + 1)$$

$$\frac{-x}{x^2 - 6x + 5} + \frac{-x - 1}{x^2 - 10x + 25}$$

$$\frac{4}{6\sqrt{3}}$$

$$\sqrt[3]{6x - 4} = \sqrt[3]{5\mathrm{x} + 8}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Making Leonardo's math…

$5280 + 1732 =$

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Obsolete

What if we designed our math curriculum around spreadsheets and…

And functional thinking

HARVARD COLLEGE

**Handbook for Students**

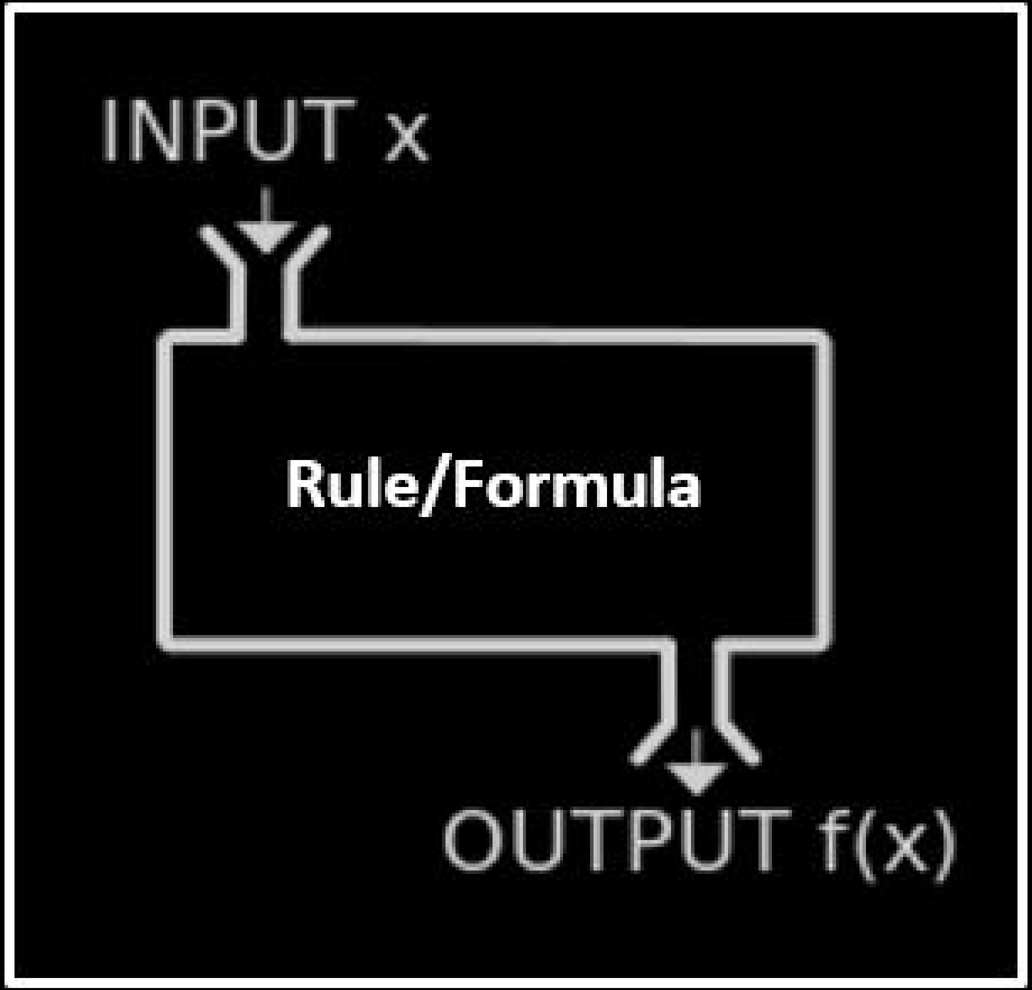
Functions are…

Perhaps the most important concept of mathematics is that of function, which provides us with the means to study dependence and change.

*Professor Peter Kronheimer, Director of Undergraduate Studies (2013-14)*

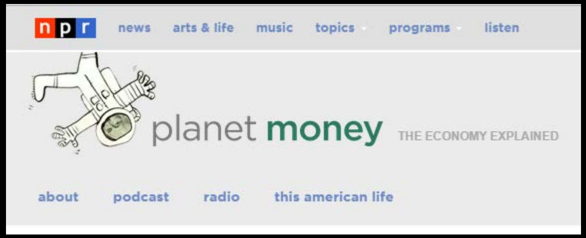
Spreadsheets are function machines with
Inputs
Outputs
Rules

Function



“Spreadsheets have left us in a different world, though. It's a world where we are constantly asking what if. And by we, I mean not just accountants and people on Wall Street. Like, all of us - me more than I would like. It's gone way beyond spreadsheets. It's like, what if I flew Thursday instead of Friday? What if I took 78 instead of Route 280? Where is traffic better? What if I stopped exercising? What if I ate more vegetables?”

DAVID KESTENBAUM HOST -- NPR Planet Money Feb 25, 2015
http://www.npr.org/blogs/money/2015/02/25/389027988/episode-606-spreadsheets

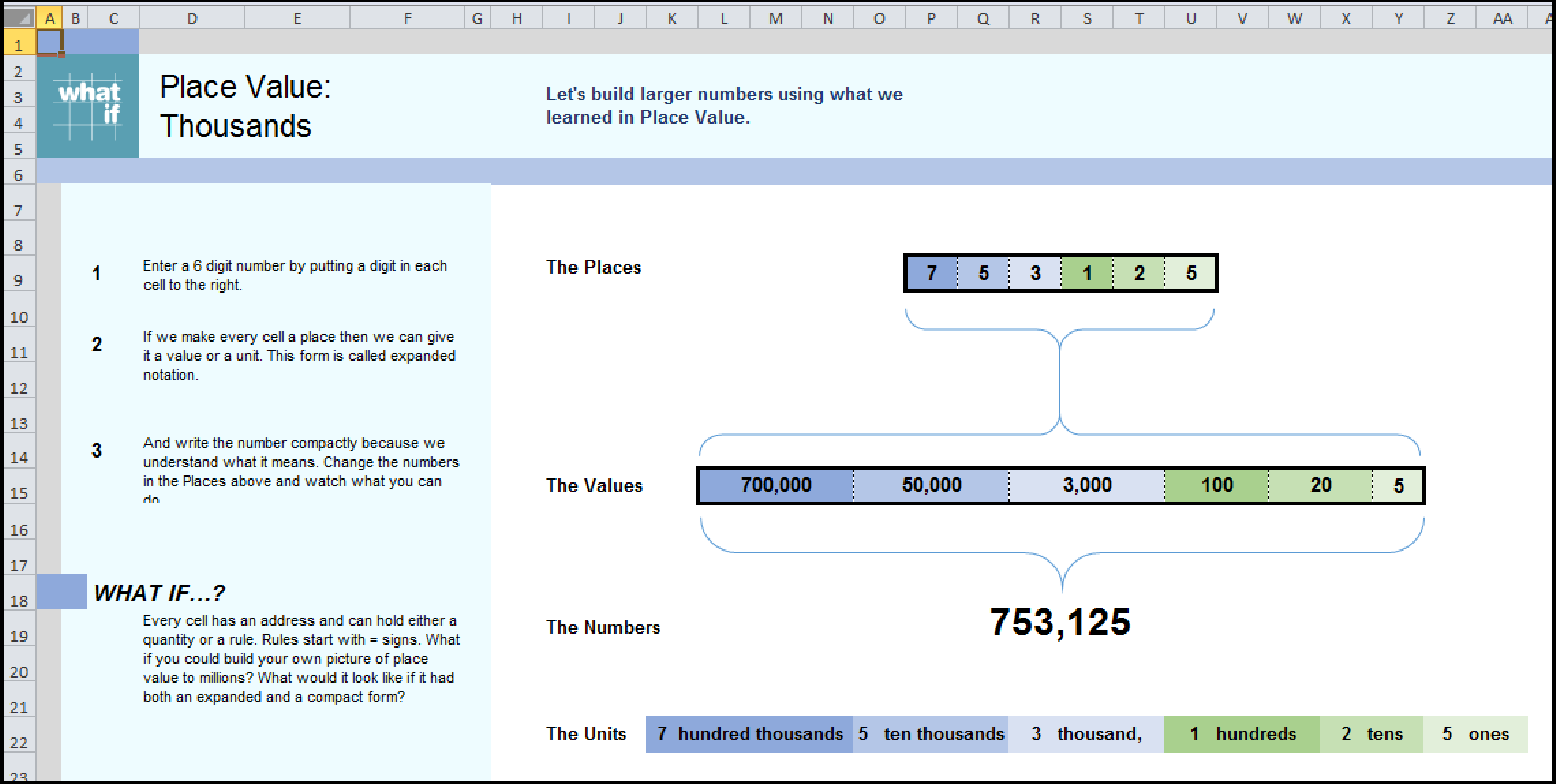


*Examples*

# Spreadsheet Lab Examples

- Place Value 1,000’s – Place Value, large numbers, powers of 10 – (MP6 Precision)
- Solving Equations – Dynamic graphing, functions – (MP5 Appropriate tools)
- Magic Rectangle – Factors, times table fluency, commutativity – (MP3 Conjectures)
- Peter’s Taxi – making problem solving and algebra concrete – (MP4 Modeling)
- Drawing Triangles – Spreadsheets can do interesting things – (MP7 Structure)
- Introducing Spreadsheets – Input, output, rules, functions – (MP2 Reasoning)
- Pascal’s Triangle – Functions, patterns, sequences – (MP8 Regularity)
- Projectile Motion – a Lab in progress/our process – (MP1 Make sense of problems)
- Composition of Functions– Operations with Functions, patterns– (MP7 Structure)

- Place Value 1,000's – Place Value, large numbers, powers of 10 – (MP6 Precision)



- Solving Equations – Dynamic graphing, functions – (MP5 Appropriate tools)

what if

## Solving Equations

Setting two functions equal to each other gives us an equation. If we graph those functions we can locate the "solutions" to that equation, the point or points where the functions intersect.

1. Consider the equation **2x+5=1**. What if we looked at each side of the equation as individual functions? Can this lead to a solution of the equation?
2. Compare the output columns of the two functions that make up this equation. **What x value produces the same output value for both functions?** What do you notice about the graphs of these functions?
3. Have you identified the solution to this equation from the tables and the graph? Check your solution algebraically?
4. Change the parameters **$m_1$, $m_2$**, ... to create different equations. Find solutions to these equations from the tables and graphs? Solve them algebraically to confirm your solutions. Did you get the same answer both ways?
5. Do the graphs always intersect at the value of x that is the solution to the equation? Why or why not?

***WHAT IF...?***

What if the graphs of the two functions do not intersect? Does this mean there is no solution? How do the tables and equations help you decide?

$$f_1(x)=m_1x+b_1 \qquad f_2(x)=m_2x+b_2$$

*f1(x)=0x+1* *f2(x)=2x+5*

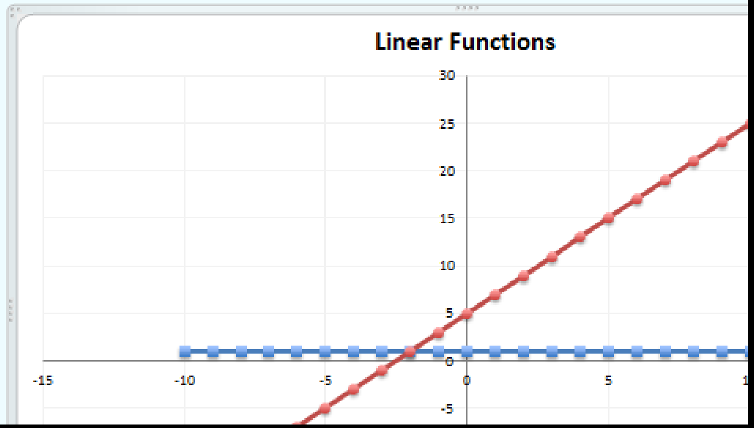
**Coefficients**

| | |
|---|---|
| $m_1$ | 0 |
| $b_1$ | 1 |

| | |
|---|---|
| $m_2$ | 2 |
| $b_2$ | 5 |

**Ordered Pairs**

| x | $f_1(x)$ | $f_2(x)$ |
|---|---|---|
| -10 | 1 | -15 |
| -9 | 1 | -13 |
| -8 | 1 | -11 |
| -7 | 1 | -9 |
| -6 | 1 | -7 |
| -5 | 1 | -5 |
| -4 | 1 | -3 |
| -3 | 1 | -1 |
| -2 | 1 | 1 |
| -1 | 1 | 3 |
| 0 | 1 | 5 |
| 1 | 1 | 7 |



- Magic Rectangle – Factors, times table fluency, commutativity – (MP3 Conjectures)

what if

# The Magic Rectangle

If you draw a rectangle on a multiplication table, will the products of opposite corners always be equal to each other?

1 Make the multiplication table on this grid without disturbing any colors by copy and pasting just the formulas (the rules).

2 Find the products of the opposite corners of this rectangle. Use the table on the right to do the computation using rules. I gave you a start, now fill in the rest.

3 Create another rectangle and try this again. Add this to the table on the right.

4 Does the size of the rectangle or its shape make any difference? Do you think this pattern is true for any rectangle you can draw?

5 Why? If you want a hint fill in the Table of Factors for each of the rectangles you tried.

## *WHAT IF...?*

Does this pattern work for every multiplication table you can make? Does it work for an odd number times table for example? Or does it work for an times table that goes to 25*25?

### Multiplication Table

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | | | | | | | | | | | | |
| 11 | | | | | | | | | | | | |
| 10 | | | | | | | | | | | | |
| 9 | | | | | | | | | | | | |
| 8 | | | | | | | | | | | | |
| 7 | | | | | | | | | | | | |
| 6 | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | |
| 4 | | | | | | | | | | | | |
| 3 | | | | | | | | | | | | |
| 2 | | | | | | | | | | | | |
| 1 | | | | | | | | | | | | |
| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

### Table of Products

| Left | Right | Product | | Left | Right | Product |
|---|---|---|---|---|---|---|
| 24 | 27 | | = | | | |
| | | | = | | | |
| | | | = | | | |
| | | | = | | | |
| | | | = | | | |

### Table of Factors

- Magic Rectangle – Factors, times table fluency, commutativity – (MP3 Conjectures)

# The Magic Rectangle

what if

**If you draw a rectangle on a multiplication table, will the products of opposite corners always be equal to each other?**

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**5** Why? If you want a hint fill in the Table of Factors for each of the rectangles you tried.

***WHAT IF…?***

**Multiplication Table**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **12** | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |
| **11** | 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| **10** | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| **9** | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| **8** | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| **7** | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| **6** | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| **5** | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| **4** | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| **3** | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| **2** | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| **1** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| * | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |

**Table of Products**

| Left | Right | Product | | Left | Right | Product |
|---|---|---|---|---|---|---|
| 24 | 27 | **648** | = | 12 | 54 | **648** |
| | | | = | | | |
| | | | = | | | |
| | | | = | | | |
| | | | = | | | |

**Table of Factors**

| | | | |
|---|---|---|---|
| 4 | 6 | 9 | 3 |
| | | | |
| | | | |
| | | | |
| | | | |

| | | | |
|---|---|---|---|
| 4 | 3 | 9 | 6 |
| | | | |
| | | | |
| | | | |
| | | | |

# Magic Rectangles

PARCC Partnership for Assessment of Readiness for College and Careers

Published on *PARCC* (http://www.parcconline.org)

Home > Grade 3 > Grade 3 Mathematics (Fluency)

## Grade 3 Mathematics (Fluency)

**Sample Item**

**Click on all the equations that are true.**

- ☐ 8 x 9 = 81
- ☐ 7 x 5 = 25
- ☐ 49 ÷ 7 = 56 ÷ 8
- ☐ 54 ÷ 9 = 24 ÷ 6
- ☐ 8 x 3 = 4 x 6

- Peter's Taxi – making problem solving and algebra concrete – (MP4 Modeling)

*If you travel by taxi in Reno, you pay a fixed fare of $1.90 per ride, plus $1.60 per mile traveled. If you travel by taxi in Denver, you pay a fixed fare of $1.50 per ride, plus $.25 per 1/10th mile traveled. On a recent trip, a taxi ride to the airport in Denver cost $12.20 more than a taxi ride to the airport in Reno. If the number of miles traveled to the airports in both cities is the same, find the distance.*

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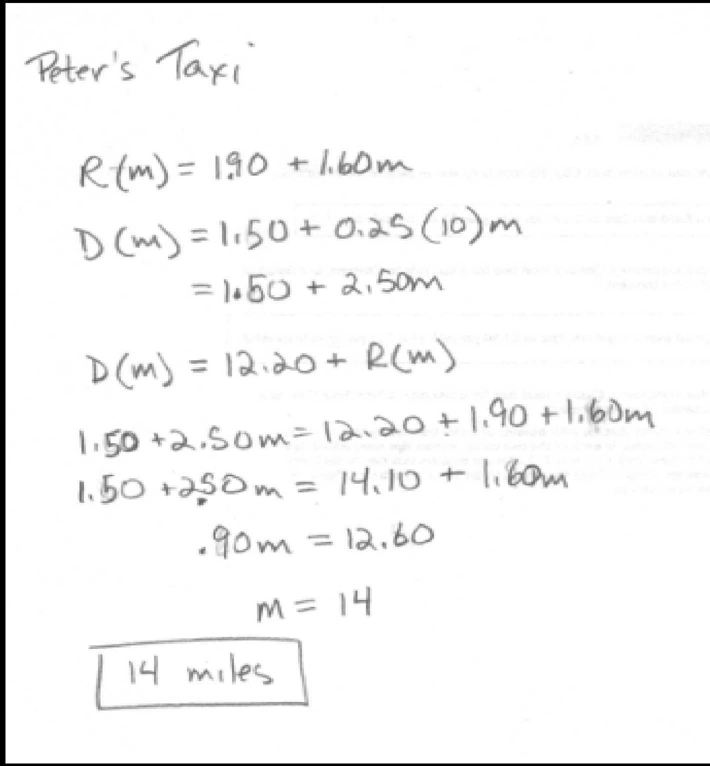
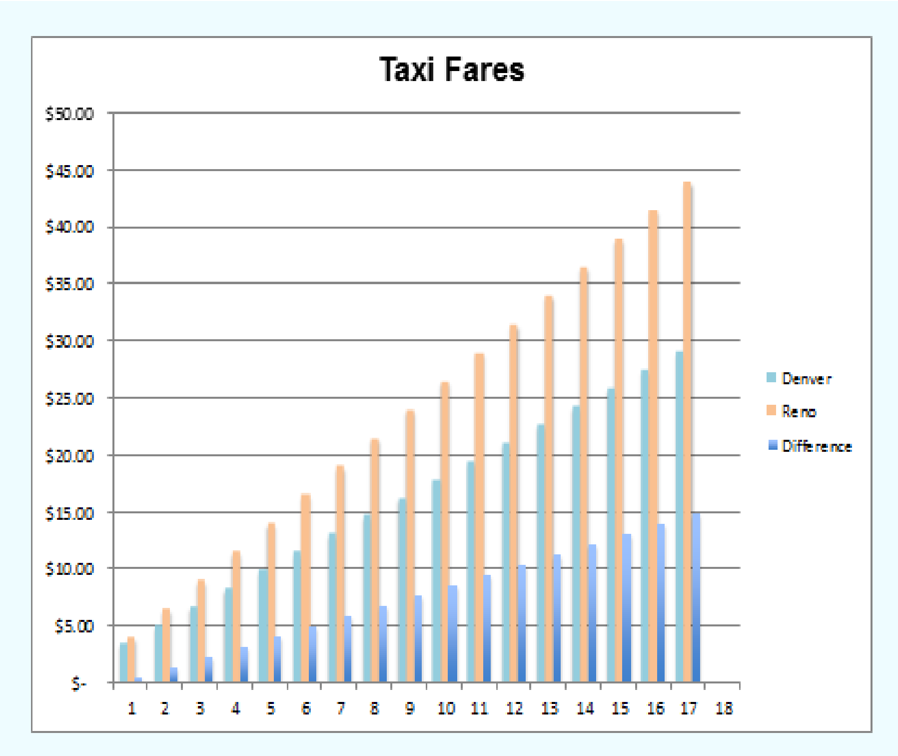


- Peter's Taxi – making problem solving and algebra concrete – MP 4 (Modeling)

| | Reno | Denver |
|---|---|---|
| Fixed Fare | $ 1.90 | $ 1.50 |
| Variable Fare | $ 1.60 | $ 2.50 |

| Distance | Reno | Denver | Difference |
|---|---|---|---|
| 1 | $ 3.50 | $ 4.00 | $ 0.50 |
| 2 | $ 5.10 | $ 6.50 | $ 1.40 |
| 3 | $ 6.70 | $ 9.00 | $ 2.30 |
| 4 | $ 8.30 | $ 11.50 | $ 3.20 |
| 5 | $ 9.90 | $ 14.00 | $ 4.10 |
| 6 | $ 11.50 | $ 16.50 | $ 5.00 |
| 7 | $ 13.10 | $ 19.00 | $ 5.90 |
| 8 | $ 14.70 | $ 21.50 | $ 6.80 |
| 9 | $ 16.30 | $ 24.00 | $ 7.70 |
| 10 | $ 17.90 | $ 26.50 | $ 8.60 |
| 11 | $ 19.50 | $ 29.00 | $ 9.50 |
| 12 | $ 21.10 | $ 31.50 | $ 10.40 |
| 13 | $ 22.70 | $ 34.00 | $ 11.30 |
| 14 | $ 24.30 | $ 36.50 | $ 12.20 |
| 15 | $ 25.90 | $ 39.00 | $ 13.10 |
| 16 | $ 27.50 | $ 41.50 | $ 14.00 |
| 17 | $ 29.10 | $ 44.00 | $ 14.90 |
| 18 | | | |



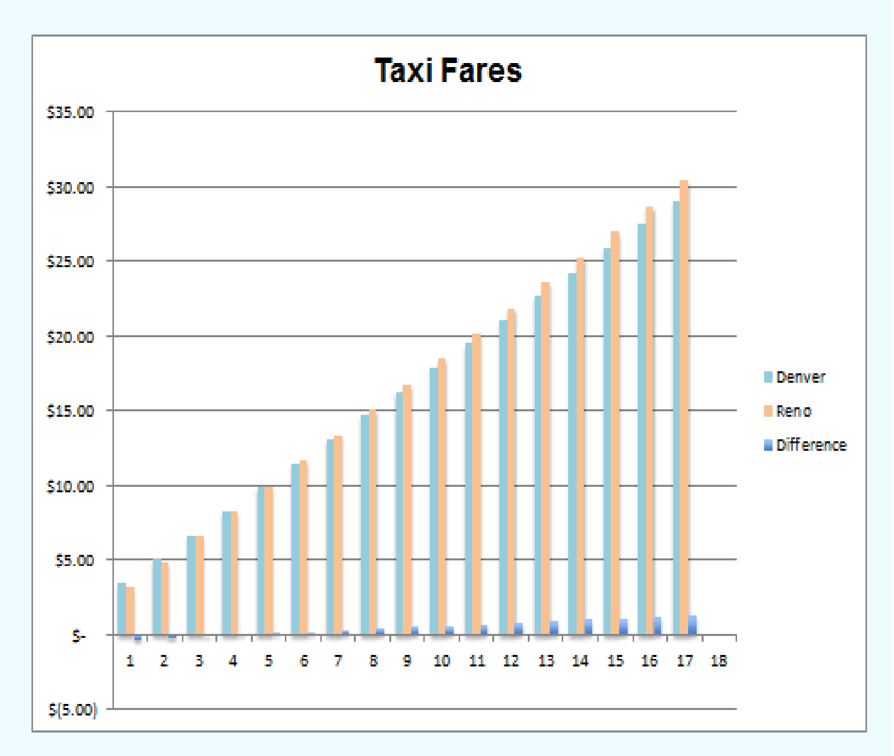
- Peter’s Taxi – making problem solving and algebra concrete – MP 4 (Modeling)

| | Reno | Denver |
|---|---|---|
| Fixed Fare | $ 1.90 | $ 1.50 |
| Variable Fare | $ 1.60 | $ 1.70 |

| Distance | Reno | Denver | Difference |
|---|---|---|---|
| 1 | $ 3.50 | $ 3.20 | $ (0.30) |
| 2 | $ 5.10 | $ 4.90 | $ (0.20) |
| 3 | $ 6.70 | $ 6.60 | $ (0.10) |
| 4 | $ 8.30 | $ 8.30 | $ - |
| 5 | $ 9.90 | $ 10.00 | $ 0.10 |
| 6 | $ 11.50 | $ 11.70 | $ 0.20 |
| 7 | $ 13.10 | $ 13.40 | $ 0.30 |
| 8 | $ 14.70 | $ 15.10 | $ 0.40 |
| 9 | $ 16.30 | $ 16.80 | $ 0.50 |
| 10 | $ 17.90 | $ 18.50 | $ 0.60 |
| 11 | $ 19.50 | $ 20.20 | $ 0.70 |
| 12 | $ 21.10 | $ 21.90 | $ 0.80 |
| 13 | $ 22.70 | $ 23.60 | $ 0.90 |
| 14 | $ 24.30 | $ 25.30 | $ 1.00 |
| 15 | $ 25.90 | $ 27.00 | $ 1.10 |
| 16 | $ 27.50 | $ 28.70 | $ 1.20 |
| 17 | $ 29.10 | $ 30.40 | $ 1.30 |
| 18 | | | |

*WHAT IF? ..........*

I would try to make the taxi fares in Denver close to those in Reno by adjusting the Fixed Fare and/or the Variable Fare so that the total cost was about the same as in Reno.



**17** Quinn works in Chicago and in New York City. He travels by taxi in each of the two cities.

| |
|---|
| In Chicago, he pays a fixed taxi fare of \$1.90 per ride, plus \$1.60 per mile traveled. |

a. Write an equation that expresses *f*, Quinn's total fare for a taxi ride in Chicago, as a function of *m*, the number of miles traveled.

| |
|---|
| In New York City, Quinn pays a fixed taxi fare of \$1.50 per ride, plus 25¢ per $\frac{1}{10}$ mile traveled. |

b. Write an equation that expresses *f*, Quinn's total fare for a taxi ride in New York City, as a function of *m*, the number of miles traveled.

c. On a recent trip Quinn noticed that the total number of miles traveled by taxi from the airport to the hotel was the same in each of the two cities. Before tips were added, his taxi fare to the hotel in New York City was \$12.20 more than his taxi fare to the hotel in Chicago. What was the distance from the airport to the hotel in each city? Show or explain how you got your answer.

- Drawing Triangles – Spreadsheets can do interesting things – (MP7 Structure)

what if

# Drawing Triangles

If you want to draw a geometric shape using a spreadsheet you can always use the drawing tools, but they are just drawing tools. They are not dynamic, you can't control them. How would you draw a triangle on a spreadsheet using functions?

1 We can use a graph. And since a triangle has 3 linear sides we would need 3 linear functions to graph each side.

2 I set the spreadsheet up for you with 3 linear functions -- their formulas and tables. I have graphed them as a scatterplot. Play with the slopes and y-intercepts to get familiar with them.

3 Create a triangle by changing the parameters in the functions. How would you classify your triangle? (**acute, obtuse, right, scalene, isosceles, equilateral**).

4 Change your functions to make as many of these other kinds of triangles as you can.

***WHAT IF…?***

What if you wanted to just show the triangle itself without the extended sides? What would have to do? Show an example.

$$f(x)=m_1x+b_1$$

$f(x)=3x+1$

$$g(x)=m_2x+b_2$$

$f(x)=3x+8$

$$h(x)=m_3x+b_3$$

$h(x)=3x+-6$

| | |
|---|---|
| $m_1$ | 3 |
| $b_1$ | 1 |

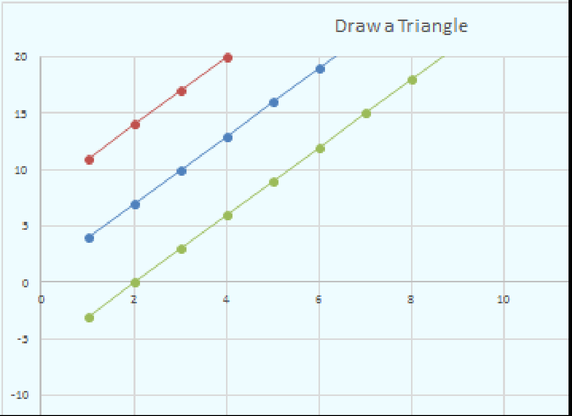
| Input x | Output f(x) |
|---|---|
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |
| 5 | 16 |
| 6 | 19 |
| 7 | 22 |
| 8 | 25 |
| 9 | 28 |
| 10 | 31 |

| | |
|---|---|
| $m_2$ | 3 |
| $b_2$ | 8 |

| Input x | Output g(x) |
|---|---|
| 1 | 11 |
| 2 | 14 |
| 3 | 17 |
| 4 | 20 |
| 5 | 23 |
| 6 | 26 |
| 7 | 29 |
| 8 | 32 |
| 9 | 35 |
| 10 | 38 |

| | |
|---|---|
| $m_3$ | 3 |
| $b_3$ | -6 |

| Input x | Output h(x) |
|---|---|
| 1 | -3 |
| 2 | 0 |
| 3 | 3 |
| 4 | 6 |
| 5 | 9 |
| 6 | 12 |
| 7 | 15 |
| 8 | 18 |
| 9 | 21 |
| 10 | 24 |



# • Drawing Triangles – Spreadsheets can do interesting things – (MP7 Structure)

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***WHAT IF…?***

What if you wanted to just show the triangle itself without the extended sides? What would have to do? Show an example.

$$f(x)=m_1x+b_1$$

*f(x)=3x+1*

| | |
|---|---|
| $m_1$ | 3 |
| $b_1$ | 1 |

| Input | Output |
|---|---|
| x | f(x) |
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |
| 5 | 16 |
| 6 | 19 |
| 7 | 22 |
| 8 | 25 |
| 9 | 28 |
| 10 | 31 |

$$g(x)=m_2x+b_2$$

*f(x)=-3x+8*

| | |
|---|---|
| $m_2$ | -3 |
| $b_2$ | 8 |

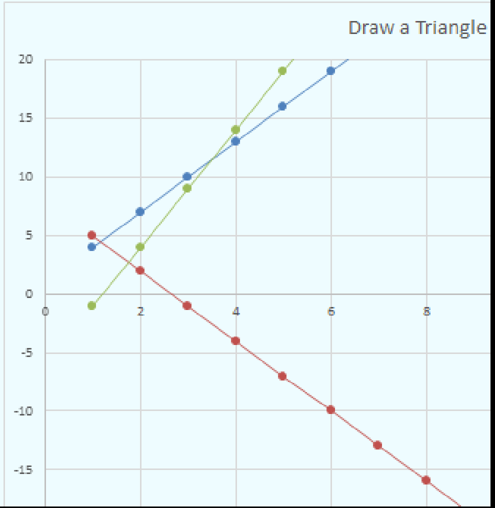
| Input | Output |
|---|---|
| x | g(x) |
| 1 | 5 |
| 2 | 2 |
| 3 | -1 |
| 4 | -4 |
| 5 | -7 |
| 6 | -10 |
| 7 | -13 |
| 8 | -16 |
| 9 | -19 |
| 10 | -22 |

$$h(x)=m_3x+b_3$$

*h(x)=5x+-6*

| | |
|---|---|
| $m_3$ | 5 |
| $b_3$ | -6 |

| Input | Output |
|---|---|
| x | h(x) |
| 1 | -1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 14 |
| 5 | 19 |
| 6 | 24 |
| 7 | 29 |
| 8 | 34 |
| 9 | 39 |
| 10 | 44 |



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*f(x)=3x+1*

$g(x)=m_2x+b_2$

*f(x)=-3x+8*

$h(x)=m_3x+b_3$

*h(x)=5x+-6*

| | |
|---|---|
| $m_1$ | 3 |
| $b_1$ | 1 |

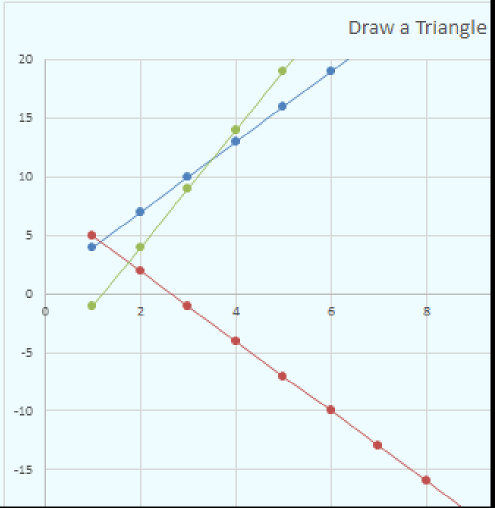
| Input x | Output f(x) |
|---|---|
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |
| 5 | 16 |
| 6 | [illegible] |
| 7 | 22 |
| 8 | 25 |
| 9 | 28 |
| 10 | 31 |

| | |
|---|---|
| $m_2$ | -3 |
| $b_2$ | 8 |

| Input x | Output g(x) |
|---|---|
| 1 | 5 |
| 2 | 2 |
| 3 | -1 |
| 4 | -4 |
| 5 | -7 |
| 6 | -10 |
| 7 | -13 |
| 8 | -16 |
| 9 | -19 |
| 10 | -22 |

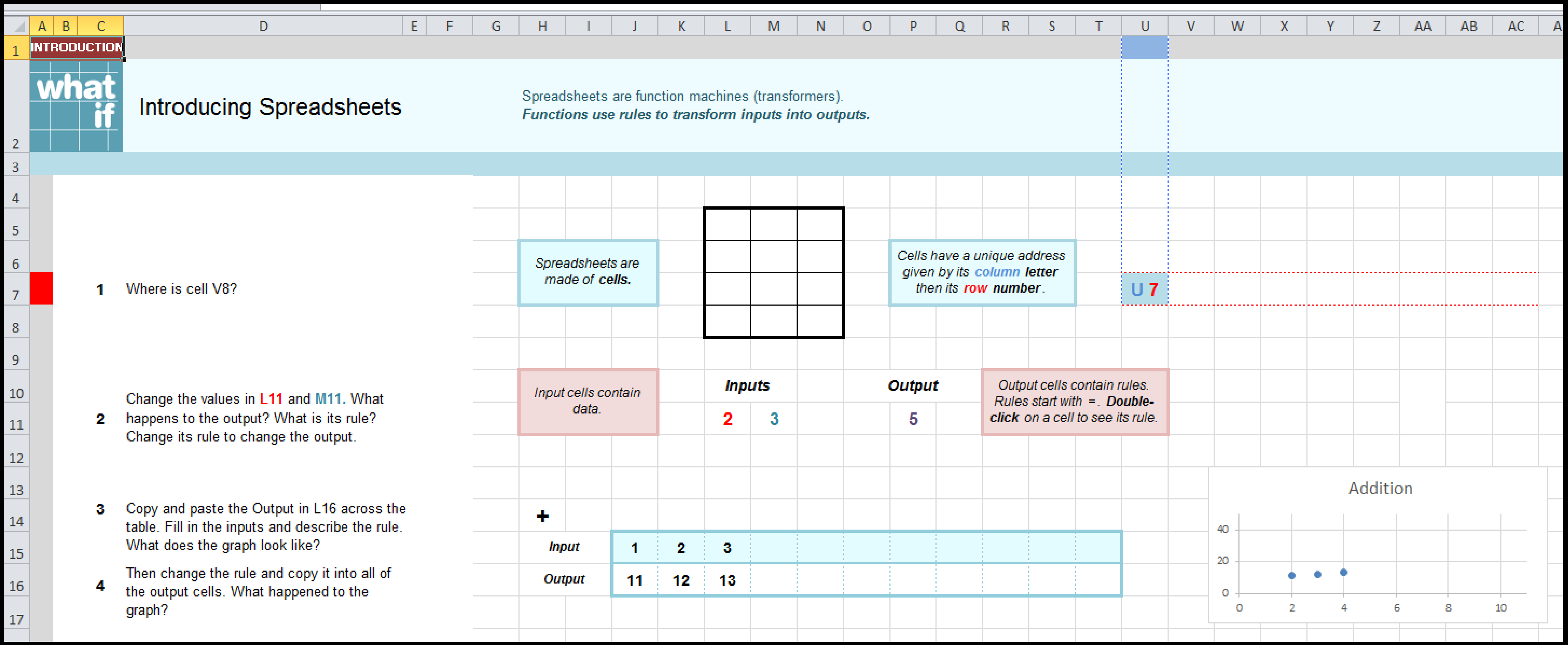
| | |
|---|---|
| $m_3$ | 5 |
| $b_3$ | -6 |

| Input x | Output h(x) |
|---|---|
| 1 | [illegible] |
| 2 | 4 |
| 3 | 9 |
| 4 | 14 |
| 5 | 19 |
| 6 | 24 |
| 7 | 29 |
| 8 | 34 |
| 9 | 39 |
| 10 | 44 |

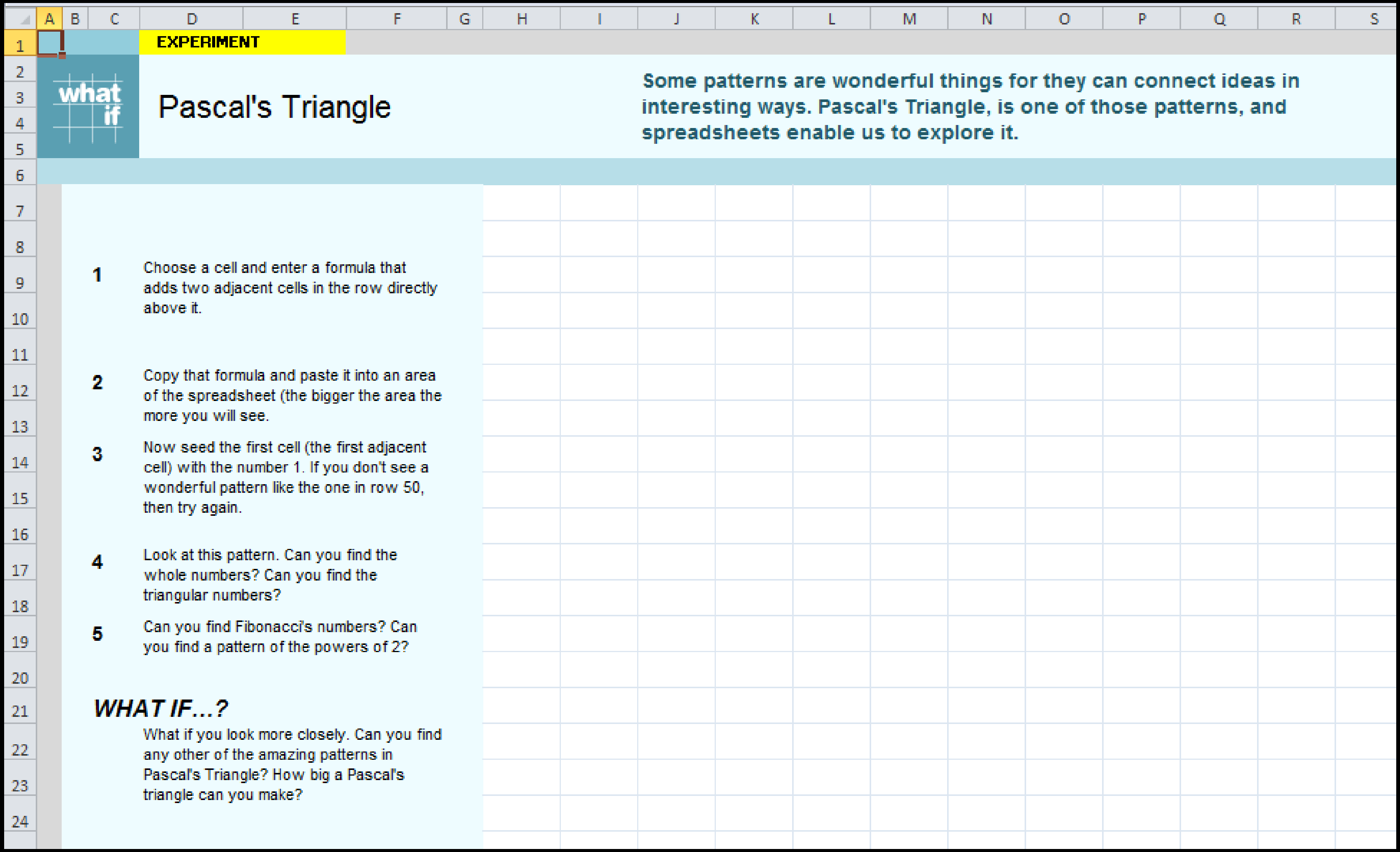



What mathematics do you see…?

- Introducing Spreadsheets – Input, output, rules, functions – (MP2 Reasoning)



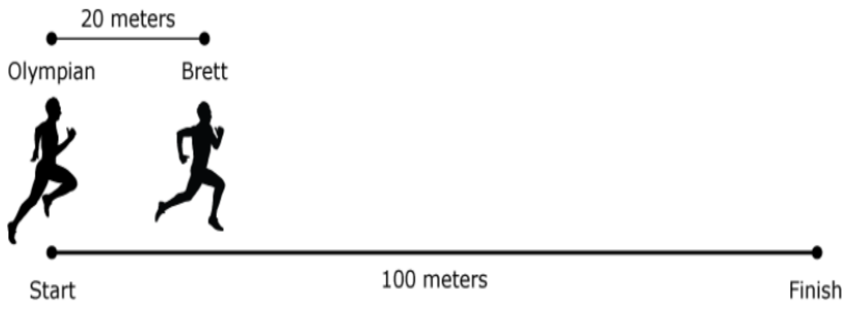
- Pascal’s Triangle – Functions, patterns, sequences – (MP8 Regularity)



- Projectile Motion – a Lab in progress/our process – (MP1 Make sense of problems)

# PARCC Sample Set HS Math -- http://parcc.pearson.com/sample-items/

Brett is on the high school track team and his coach surprises the team by having an Olympic track champion attend a practice. The Olympian challenges Brett to a 100-meter race. To make the race more interesting, the Olympian will not start the race until Brett reaches the 20 meter mark. Brett's average time in the 100-meter race is 12 seconds, while the Olympian's average time is 10 seconds. Assume that Brett and the Olympian run at a constant speed throughout the race.




**Part A**

Based on each of the runner's average times, write an equation for each person that describes the relationship between his distance from the starting line, in meters, and time, in seconds.

**Part B**

Based on your equations in Part A, who will win the race and by how much? Justify your answer.

- Composition of Functions– Operations with Functions, patterns– (MP7

# what if

## Composition of Functions

You can take the outputs of one function and use them as the inputs of another function to create a new function. This is a powerful and wonderful idea!

1 Suppose you have two functions, a linear function (pink) and a power function (blue).

2 Complete the table for each function **f(x)** and **g(x)**. Graph them on the same graph. To see the patterns more easily, use the same values for **x** .

3 Take the output values for **f(x)** and use them for the input **values** for **g(x)**. Call this new function **h(x)** and complete the 3rd table to create it and preserve the original **g(x)**. See any patterns?

4 Graph **h(x)**. What does this graph look like? How does this graph compare to **f(x)** and **g(x)**?

5 Change the parameters **m, b, a**, and/or **p**. What patterns do you see?

***WHAT IF…?***

What if we were to change the order of **h(x)** and take **f(g(x))** instead of **g(f(x))**? In effect we can ask if taking a function of a function is commutative? Well is it?

$$f(x)=mx+b$$

f1(x)=3x+5

$$g(x)=ax^p$$

g(x)=1x^2

$$h(x)=g(f(x))$$

| | |
|---|---|
| m | 3 |
| b | 5 |

| Input | Output |
|---|---|
| x | f(x) |
| | |

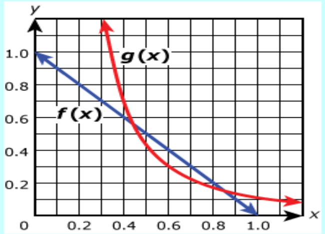
| | |
|---|---|
| a | 1 |
| p | 2 |

| Input | Output |
|---|---|
| x | g(x) |
| | |

| Input | Output |
|---|---|
| | h(x) |
| | |

# Function Labs

The functions $f(x) = 1 - x$ and $g(x) = \frac{0.11}{x^2}$ are defined for all values of $x > 0$. The graphs are shown in the coordinate plane.




Part A

Explain how you can use the graph to find the solution(s) of the equation $f(x) = g(x)$. In your answer, provide the approximate value(s) of the solution(s).

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Part B

Write the value(s) of $f(x)$ when $x$ equals the solution(s) from Part A.

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Part C

Let the function $h(x)$ be defined as $h(x) = f(x) - g(x)$.

What are the coordinates of the point(s) on the graph of $h(x)$ when $x$ equals the solution(s) from Part A? Explain your reasoning.

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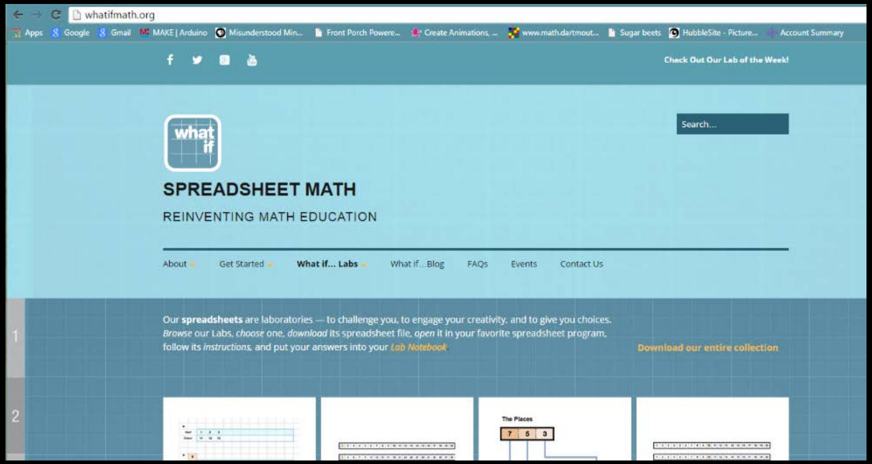
Thank You




We would love your feedback!

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